On 2D Drift Wave-Zonal Flow Interactions with Pseudo-Spectral Simulations

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ABSTRACT

Plasmas in fusion devices such as tokamaks undergo a chaotic process called turbulence which affects their performance by controlling the rate at which particles leak out of the magnetic trap. Understanding the dynamics of turbulent transport in tokamaks and its features such as zonal flows and drift waves, is an extensive field of study. We have developed a code for pseudo-spectral simulations of 2D drift wave turbulence of tokamak plasma based on the Dedalus framework (dedalus-project.readthedocs.io),an open source MPI parallelized python code. We will first present the results of the simulations for a hydrodynamic vortex merger test case in the vorticity formulation of the 2D Navier Stokes Equations with viscosity. We also show the results for the evolution of the Hasegawa-Mima equations and the Terry-Horton equations, along with their properties such as enstrophy and energy conservation in the zero-viscosity case. Self-generated zonal flows can strongly reduce turbulence. We will study the impact of various models of zonal flows on the nonlinear saturation level of turbulence, including a model of the strong neoclassical shielding of zonal flows in tokamaks. This code will be made publicly available.

1. INTRODUCTION

One of the main goals in the field of plasma physics is producing nuclear fusion power. This is done using various techniques, of which magnetic confinement seems promising. Magnetic confinement entails confining quasi-neutral plasmas in a container, such as tokamaks, using magnetic fields so that they form a trap to increase the probability of collision as well as improve the confinement time. Tokamak plasmas constantly undergo turbulence throughout their motion. The Drift-Wave (DW) turbulence, in particular, is a major turbulence that results from primary and secondary DW instabilities that form when the electrons and ions move at different speed or 'drift' relative to one another. They are low-frequency waves that occur in magnetised plasma with non-uniform density. An interesting feature of DWs are the spontaneous generation of vertical shear flows called zonal flows (ZFs) that affect the DW turbulence itself. In fact, below a certain threshold the zonal flows completely suppress DWs, which is widely studied phenomena known as the Dimits shift (Dimits et al. (2000)). This is mainly important in turbulence driven by ion temperature gradients (and to some extent electron temperature gradients) where the DW energy is transferred away from them to small scales as shown using gyrofluid equations, which include kinetic effects such as Landau damping in extending the fluid model (Hammett et al. (1993)). ZF suppression of DW helps regulate the temperature gradient and is thus its study is of interest in the plasma physics community. Zhu et al. (2020)

2. THE FLUID AND PLASMA EQUATIONS

2.1. *The Navier Stokes Equation*

We begin with the 2D neutral fluid equation

$$\frac{\partial \vec{v}}{\partial t} + (\vec{v}\cdot\nabla)\vec{v} = \nu\nabla^2\vec{v} - \nabla p \tag{1}$$

where ν is the viscosity causing the viscous damping term, $\vec{v} = v_x\hat{x} + v_y\hat{y}$ is the 2D velocity of the fluid, p is the scalar pressure whose gradient affects the fluid velocity in the opposite direction. We do not consider the affect of gravity here. For simplicity, we assume that the fluid is incompressible which imposes the condition

$$\nabla\cdot\ \vec{v}\ =\ 0 \tag{2}$$

We introduce the helpful quantities $\omega = (\nabla \times \vec{v}) \cdot \hat{z}$, which is the scalar vorticity, and $\psi : \nabla\psi \times \hat{z} = \vec{v}$, which is called the stream function (or stream potential). With these new definitions we recast the Navier Stokes equation and incompressibility. Consider the 2D case where the vorticity is only in the z direction. Then, the two components (x, y) of the equation would read

$$\frac{\partial v_x}{\partial t} + v_x \frac{\partial v_x}{\partial x} + v_x \frac{\partial v_x}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) \tag{3}$$

$$\frac{\partial v_y}{\partial t} + v_y \frac{\partial v_y}{\partial x} + v_y \frac{\partial v_y}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial y} + \nu \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) \tag{4}$$

Now we take the curl of the Navier Stokes equation by taking the x derivative of the y component subtracted by the y derivative of the x component from Equations (3) and (4) as given by the curl formula to yield

$$\begin{aligned} &\frac{\partial}{\partial t}\left(\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y} \right) + v_x \frac{\partial}{\partial x}\left(\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y} \right) + v_y \frac{\partial}{\partial y}\left(\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y} \right) \\ &\qquad + \left(\frac{\partial v_x}{\partial x} - \frac{\partial v_y}{\partial y} \right)\left(\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} \right) = 0 \end{aligned} \tag{5}$$

But notice that under the 2D assumption the vorticity becomes

$$\omega = \nabla \times \vec{v} = \frac{\partial v_y}{\partial x} - \frac{\partial v_x}{\partial y} \tag{6}$$

which is exactly the terms inside the parentheses occurring in Equation (5). We also have from the stream potential definition

$$v_x = \frac{\partial \psi}{\partial y} \qquad v_y = -\frac{\partial \psi}{\partial x} \tag{7}$$

along with imposing the continuity equation

$$\frac{\partial v_x}{\partial x} + \frac{\partial v_y}{\partial y} = \frac{\partial^2 \psi}{\partial x \partial y} - \frac{\partial^2 \psi}{\partial x \partial y} = 0 \tag{8}$$

to finally yield the vorticity evolution equation as

$$\frac{\partial \omega}{\partial t} + v_x \frac{\partial \omega}{\partial x} + v_y \frac{\partial \omega}{\partial y} = \nu \left(\frac{\partial^2 \omega}{\partial x^2} + \frac{\partial^2 \omega}{\partial y^2} \right) \tag{9}$$

or cast in a more compact form as it appears in the literature (Peterson & Hammett (2013))

$$\frac{\partial \omega}{\partial t} = -[\omega, \psi] + \nu \nabla^2 \omega \tag{10}$$

where the Poisson bracket term is the nonlinear advection term defined as follows

$$[\omega, \psi] = \frac{\partial \omega}{\partial x}\frac{\partial \psi}{\partial y} - \frac{\partial \omega}{\partial y}\frac{\partial \psi}{\partial x} \tag{11}$$

along with the incompressibility condition after substituting the stream potential for the velocity

$$\nabla^2 \psi = -\omega \tag{12}$$

Equation (10) is the Navier Stokes equation in the vorticity formulation, while (12) is the incompressibility condition which relates the stream function and the vorticity.

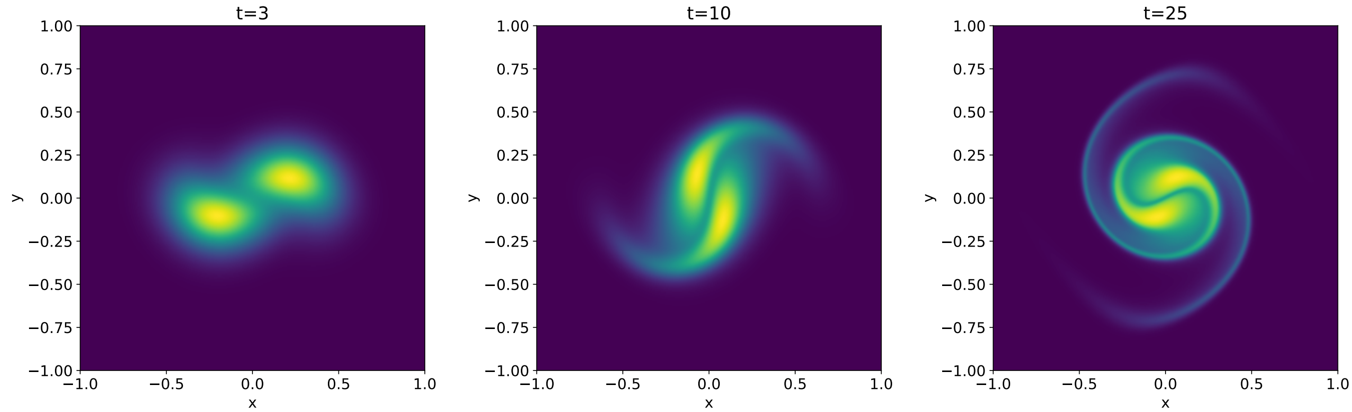

Figure 1: Hydrodynamic Equation evolution time snapshots.

2.2. *The Hasegawa-Mima Equation*

The paradigm equation that models the DW turbulence in a 2D slice in the tokamak is the Hasegawa-Mima equation (HWE) (Hasegawa & Mima (1977)), that takes into account a vertical drift of the ion density from electric potential fluctuations. Since, it is only in 2D it assumes no toroidal variations of the potential. It is given as follows

$$\frac{\partial \xi}{\partial t} + \boldsymbol{v} \cdot \nabla \xi - \kappa \frac{\partial \psi}{\partial y} = 0 \tag{13}$$

$$\xi = \nabla^2 \psi - \psi \tag{14}$$

where ξ is the ion gyrocentre density, ϕ is the potential such that $v_x = -\frac{\partial \phi}{\partial y}$ $v_y = \frac{\partial \phi}{\partial x}$. An interesting property of the HME is that it conserves energy and enstrophy which are defined in a slightly modified way, respectively

$$E = \frac{1}{2} \int \left(|\nabla \psi|^2 + \psi^2 \right) d^2\boldsymbol{x} \quad Z = \frac{1}{2} \int \left(|\nabla^2 \psi|^2 + |\psi|^2 \right) d^2\boldsymbol{x} \tag{15}$$

where E contains the kinetic first term and the potential second term

2.3. *The Terry Horton Equation*

The HME is not the complete picture of the turbulence since it does not have primary instabilities and is reversible. By includes a linear stability through a phase difference that modifies the Poisson equation as follows Terry & Horton (1982)Terry & Horton (1983)

3. THE MODIFIED TERRY HORTON EQUATION WITH NEOCLASSICAL SHIELDING

4. NUMERICAL RESULTS

5. CONCLUSION

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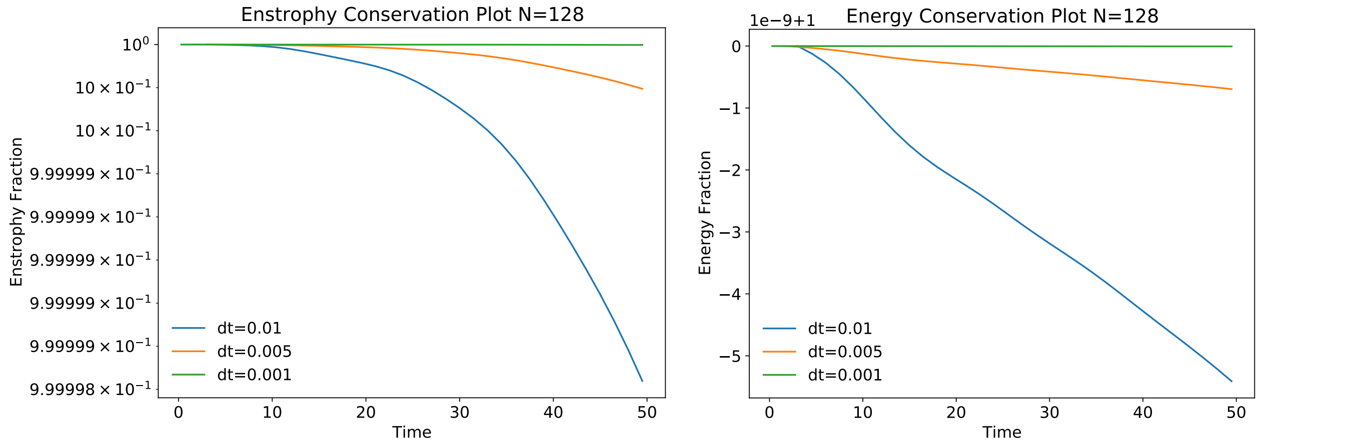

Figure 2: Hydrodynamic (Left) Enstrophy conservation plot showing variation with time step and (Right) Energy conservation plot showing variation with time step.

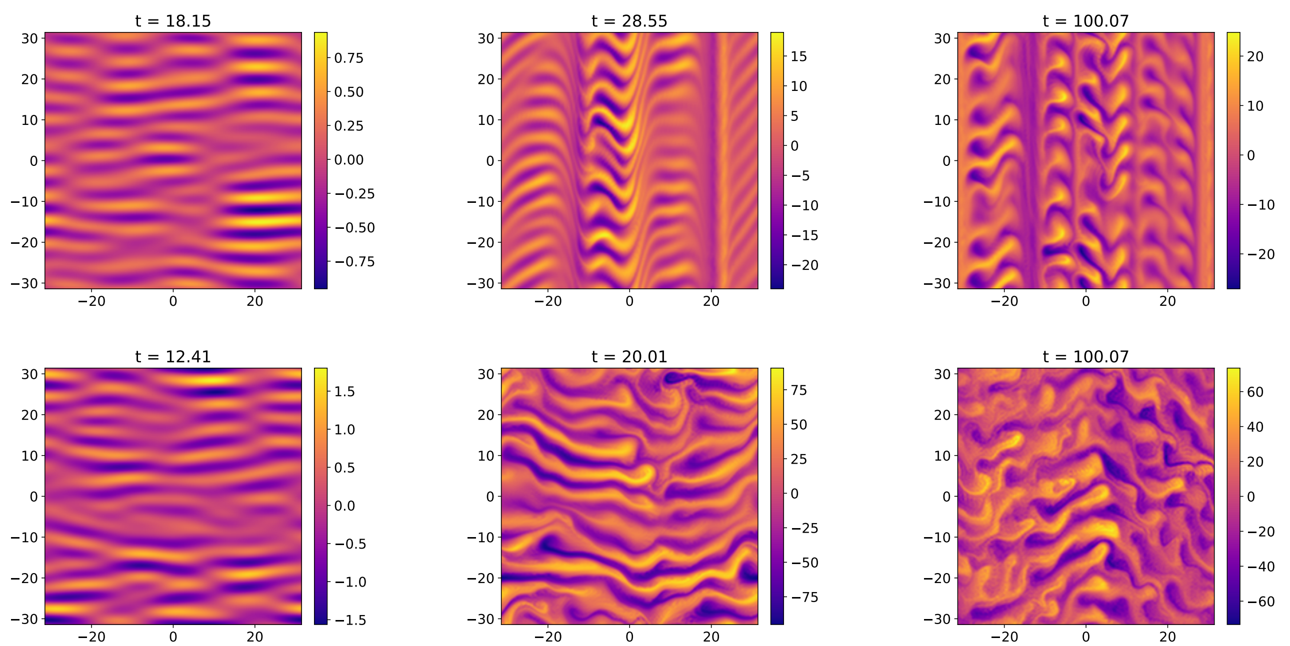

Figure 3: MTHE with neoclassical shielding time snapshots for (Top) $\kappa = 6$, $s = 2$ and (Bottom) $\kappa = 7$, $s = 10$

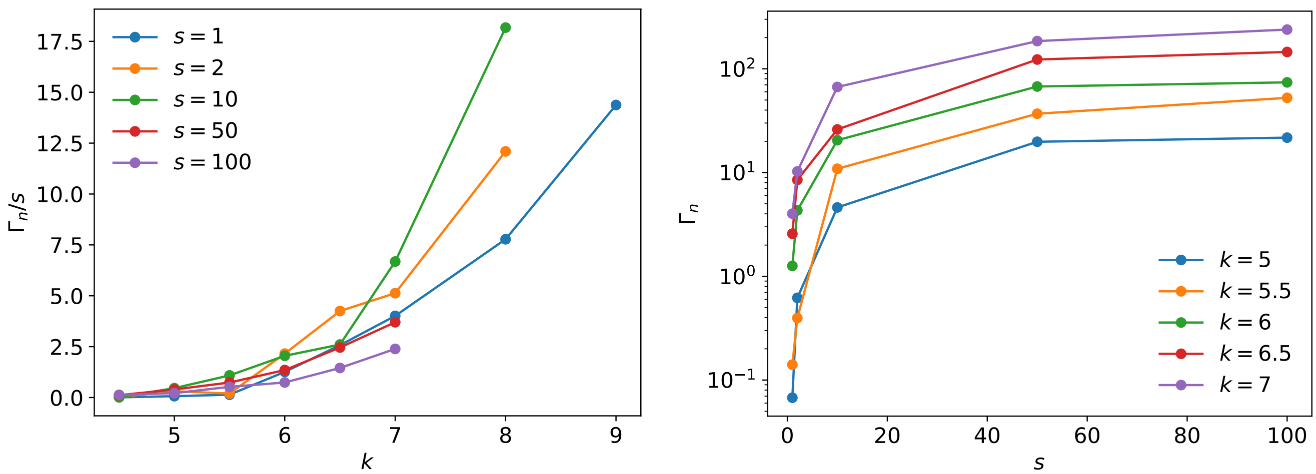

Figure 4: Caption