Princeton University
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Certificate in Applied and Computational Mathematics — Final Paper

Black Holes Stimulate Higgs Vacuum Decay

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Abstract

In this paper, we provide evidence that black holes with masses around $\sim 0.1\,\mathrm{kg}$ severely enhance the Higgs vacuum decay rate to the true vacuum. Results from the LHC show that in flat space the lifetime of the Higgs vacuum is much longer than the estimated age of the universe. However, assuming the standard model, we find that a single primordial black hole that evaporates down to a mass of $m_{BH} \sim 0.03\,\mathrm{kg} - 0.17\,\mathrm{kg}$, could make the transition to the true vacuum almost instantaneous. The enhancement of the decay rate is caused primarily by metric distortions around the black hole, as we show that thermal and backreaction effects are negligible. Our paper thus places tension between the standard model and cosmological models which predict the existence of primordial black holes in our past lightcone.

May 11, 2015

Contents

1 Introduction

The recent results at the LHC not only confirm our understanding of the standard model, with the discovery of the Higgs boson, but also give an indication about the stability of our vacuum. If the standard model is valid up to energies greater than 10^{12}GeV, the observed masses of the Higgs boson and the top-quark indicate that the vacuum is meta-stable and that it will transition to a lower energy state in the future. Nevertheless, in flat space, the decay-time of the false vacuum is much greater than the current lifetime of the Universe [1].

While these results apply in the absence of any form of matter, in order to get a realistic estimate of the decay-time of the false vacuum, one has to analyze the behavior of the Higgs field in the presence of inhomogeneities. Intuitively, we can expect the decay-time to the true vacuum to be significantly altered by such metric inhomogeneities using the formalism developed by Coleman and de Luccia [2]. When creating a true vacuum bubble blue, the gained energy, which is proportional to the volume of the bubble, is balanced by the surface tension. In other words, the nucleation rate depends on the bubble's surface-to-volume ratio and this ratio can be highly reduced by strong gravitational effects. Gregory *et al.* [3] considered the nucleation of true vacuum bubbles around black holes, taking into account the contribution of conical singularities to the Euclidean on-shell action for the bubble-wall solution. They found that the contribution from metric effects to the vacuum decay rate is sensitive to the mass of the nucleating black holes. Burda *et al.* [4] applied this formalism to the Higgs field, concluding that potentials subject to quantum gravity corrections at small field displacements put tension between the standard model and primordial black hole physics. If such corrections do not apply for field displacements well bellow the Planck scale, Burda *et al.*'s methodology cannot be extended to the standard approximation of the effective Higgs potential.

In this paper, we provide an improved estimate of the decay rate of the Higgs vacuum by including effects near black holes that invalidate the thin wall approximation. We find a dangerous black hole mass range, $m_{BH} \sim 0.03\text{kg}{-}0.17\text{kg}$, for which the decay-time to the real vacuum is significantly reduced, leading to an almost instantaneous decay. Typical scenarios of primordial black hole creation predict masses which are much higher than the discovered dangerous mass range [5]. However, as it is expected for black holes to evaporate and probe the entire mass range smaller than their initial mass, a single primordial black hole that evaporated in our past light-cone is enough to catalyze the decay of the false Higgs vacuum. This implies a strong tension between the standard model and the current theory of primordial black holes. Specifically, our results indicate one of two possibilities. If primordial black holes have evaporated in our past vacuum, then the effective potential of the Higgs is stableized by undiscovered particles that couple to the Higgs field, making the standard model ineffective at energies higher than 10^{12}GeV. If the standard model is indeed valid up to high energies, our results imply that primordial black holes have never formed or have not yet evaporated to the dangerous mass range.

This paper is organized as follows. In Section 2, we review the derivation of the decay-time to the true vacuum for a generic scalar field and then for the Higgs field. In Section 3, we present the variational method through which we have managed to find upper-bounds for the decay time to the true Higgs vacuum in the presence of black holes and show that our result is impervious to thermal or metric backreation effects. Finally, in Section 4, we discuss further steps to generalize our result for other types of inhomogeneities, such as cosmic strings or gravitational instantons. Finally, we also discuss the implications of our results for the standard model and for black hole physics.

2 The Higgs Instability in Flat Space

In order to determine the transition rate between the false vacuum and the true vacuum, we write the transition probability between the two states as a path integral in terms of the Euclidean action $S_E[\phi(\mathbf{x},\tau)]$,

$$\langle \phi(\mathbf{x},-\infty) \,|\, \phi(\mathbf{x},0) \rangle = \int_{\phi=\phi(\mathbf{x},-\infty)}^{\phi=\phi(\mathbf{x},0)} D\phi e^{-S_E[\phi(\mathbf{x},\tau)]}, \tag{1}$$

where τ is the Euclidean time, $\phi(x,-\infty)=0$ is the field configuration corresponding to the false vacuum and $\phi(x,0)$ corresponds to the false vacuum configuration. The boundary conditions for the false vacuum configuration are given by,

$$\phi(0,0)=\phi_0, \quad \partial_r\phi(0,0)=0, \quad \partial_t\phi(0,0)=0. \tag{2}$$

Given that there exists a classical bounce solution $\phi_b(x,\tau)$ that minimizes S_E and transitions between the initial $\phi_b(x,-\infty)=\phi(x,-\infty)$ and final $\phi_b(x,0)=\phi(x,0)$ field configuration, the Euclidean action of this solution would dominate in the path integral. While in flat space we can find an exact analytic bounce solution, in curved space finding a solution is strenuous. However, any other field configuration $\tilde{\phi}_b(x,\tau)$ which respects the boundary conditions of path integral (2), will have a smaller associated Euclidean action than the actual bounce solution $\phi_b(x,\tau)$. Consequently, the action given by any other field configuration will give an upper bound on the expected transition time to the true vacuum.

In order to determine the analytic bounce solution in flat space, we use the Euclidean space-time O(4)-symmetry. Concretely, we rewrite the metric as, $ds^2=d\eta^2+\eta^2 d\Omega_3^2$, where $\eta=\sqrt{\mathbf{x}^2+\tau^2}$ and $d\Omega_3$ denotes an element of the three dimensional unit sphere.

Assuming the SM is valid up to large field displacements ($\phi \sim 10^{12}$GeV), the quartic term in the Higgs potential becomes negative and dominates over the mass term in the potential. This approximation is valid for field displacements up to the Planck scale, where higher order terms in the Higgs potential become relevant. Therefore, if we believe that the Standard Model is valid up to field displacements $10^{12}\text{GeV} \ll \phi \ll 10^{19}\text{GeV}$, we can approximate the effective Higgs potential to be $V_{\text{eff}}(\phi)=-\lambda\phi^4/4$ with $\lambda>0$. For such a potential, Lee and Weinberg [6] found the Fubini instanton solution:

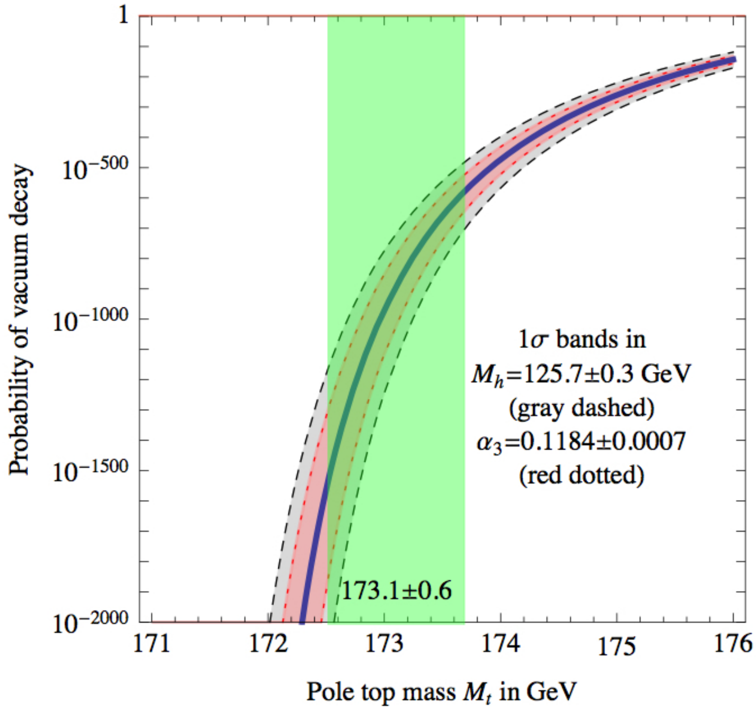

Figure 1: Dependence of the probability of vacuum decay up to current time as a function of the top-quark mass, as presented by Buttazzo *et al.*[1].

$$\phi(\eta) = \sqrt{\frac{8}{\lambda}}\frac{b}{\eta^2 + b^2}, \tag{3}$$

where b is an integration constant which characterizes the size of the instanton. The Euclidean action which gives the probability of vacuum decay can then be computed as,

$$S_E = \int d\eta \left(4\pi^2\eta^3\right)\left[-(\partial_\eta\phi)^2 - V_{\rm eff}(\phi)\right] = \frac{8\pi^2}{3\lambda}, \tag{4}$$

where the action is independent of the length-scale of the instanton, b. In the absence of gravitational perturbations, using the estimates for λ by Butazzo et al. [1], which depend on the observed mass of the top-quark, one can determine the probability of the Higgs vacuum decay rate over the lifetime of the Universe. According to their estimates, the expected lifetime for the Higgs vacuum decay is,

$$t \sim 10^{775} \text{ years}, \tag{5}$$

a time much longer than the current lifetime of the universe. At the 1-sigma level, the lifetime of the Higgs vacuum decay is roughly between $t \sim 10^{400} - 10^{1000}$years (see Figure 7 in [1]). A stable Higgs vacuum is excluded at 2-sigma level but is possible at 3-sigma level. Thus, in flat space, results from the LHC indicate that it is likely that the decay time to the real vacuum does not put any tension on the standard model.

3 The Fubini Instanton and Primordial Black Holes

While in the previous section the reviewed results indicate that the meta-stability of the Higgs vacuum does not pose any real danger to our universe, this might not be the case in curved space-time. Gregory *et al.* [3] analyzed the specific case of Coleman-de Luccia instantons in the presence of black holes and showed that conical defects led to a potential increase in the false vacuum decay rate. More recently, Burda *et al.* [4] argued that the results obtained in [3] might be extrapolated to the Higgs field, leading to tensions in the SM or in black hole physics. However, [3, 4] only considered the thin-wall bubble approximation, in which there is a discontinuous transition between the true and false vacuum. For the effective potential of the Higgs field, the energy difference between the false and real vacua is large rendering the thin-wall bubble approximation to be inaccurate. This is the reason why, Burda *et al.* [4] limited their analysis only to a set of Higgs potentials which experience strong quantum gravity corrections for small values of the field displacement, $\phi \ll M_{PL}$.

In order to analyze the realistic scenario in which the Higgs potential only has a quartic term, with λ in the range estimated by Butazzo *et al.*, we compute the action of continuous instanton solutions. Thus, to obtain realistic results for the Higgs field, we have extended the analysis of Lee and Weinberg [7] by studying Fubini instanton solutions centered around black holes. The effective action is given by,

$$S_{\text{eff}} = \int dr(4\pi r^2)d\tau\sqrt{g}\left[-g^{\mu\nu}\partial_\mu\phi\partial_\nu\phi - V_{\text{eff}}(\phi)\right] \tag{6}$$

where $g = \text{Tr } g^{\mu\nu}$ and we assume that $g^{\mu\nu}$ is given by the Schwarchild solution,

$$ds^2 = \left(1 - \frac{r_s}{r}\right)d\tau^2 + \left(1 - \frac{r_s}{r}\right)^{-1}dr^2 + r^2 d\Omega_2^2 \tag{7}$$

which implies that the equations of motion can be written as,

$$\frac{r^2(r-r_s)\lambda\phi^3(r,\tau) + r^3\partial_\tau^2\phi(r,\tau) + (2r^2 - 3rr_s + r_s^2)\partial_r\phi(r,\tau)}{r(r-r_s)^2} + \partial_r^2\phi(r,\tau) = 0 \tag{8}$$

While determining the exact solution for the Fubini instanton in the presence of black holes is difficult, one can avoid this problem by bounding the Euclidean action instead of analytically solving the field equations. In order to find such bounds, we consider field profiles which do not solve the equations of motion for the Higgs field, but only respect the appropriate boundary conditions imposed on the path integral in Eq. (1). The boundary conditions at the horizon of the black hole and at infinity are,

$$\phi(r,0) = \phi_0(r), \qquad \partial_r\phi(r_s,0) = 0, \qquad \partial_t\phi(r_s,0) = 0$$
$$\phi(r,t)\Big|_{r\to\infty} = 0, \qquad \phi(r,t)\Big|_{t\to-\infty} = 0. \tag{9}$$

The first boundary condition imposes that the we transition from the false vacuum state to some non-zero field-profile $\phi_0(r)$. Imposing that the field does not vary in position in time at the horizon surface is necessary in order to obtain a finite kinetic term in the Euclidean action. Furthermore, we impose that the field decays to zero as metric effects are negligible far away from the black hole. Thus the instanton solution in the presence of the black hole has a similar asymptotic behavior to the solution in flat-space. Finally, the last boundary condition imposes that the initial field configuration corresponds to the false vacuum state.

Given that the action is minimized by the field profile $\phi_b(r,t)$ which is the solution of the equation of motion, any other field profile $\tilde{\phi}_b(r,t)$ which respects the appropriate boundary conditions should yield a greater Euclidean action. Therefore, such a field profile will give an upper-bound on the decay time for the Higgs vacuum. In this sense, our approach is similar to the standard variational method from quantum mechanics.

We have considered different field profiles, similar to solution (6) for the Fubini instanton in flat space:

$$\tilde{\phi}_b(r,\tau) = \sqrt{\frac{8}{\lambda}}\frac{b}{b^2+(r-r_h)^2+t^2} \times$$
$$\times\left(1+\frac{\alpha b^2}{b^2+(r-r_h)^2+t^2}\right)^{\beta} \tag{10}$$

and have tuned the parameters α and β such that we obtain a Euclidean action as small as possible. As mentioned previously, the field profile that we've considered in Eq. (10), respects the boundary conditions from Eq. (9) as $r\to r_h$ and $t=0$, $r\to\infty$ and $t\to\infty$. Since the theory is scale invariant, the Euclidean action associate to field profiles $\tilde{\phi}_b(r,\tau)$ will only depend on the ratio between the black hole and instanton size, b/r_h. By rescaling the field $\tilde{\phi}_b \to \sqrt{\lambda}\tilde{\phi}_b$

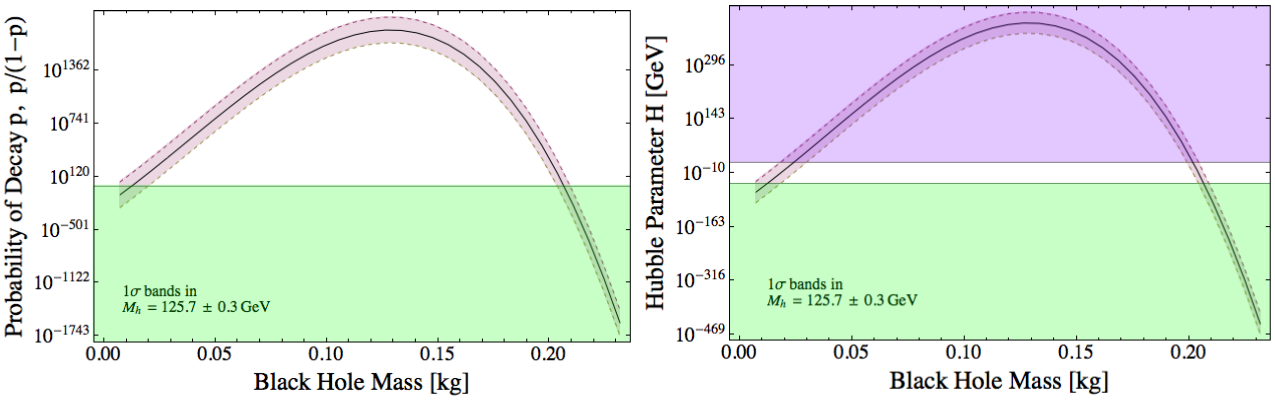

Figure 2: On the *top*, we plot the dependency of the probability of vacuum decay up to current time as a function of the primordial black hole mass. Note that for the appropriate mass range the transition becomes almost certain. On the *bottom*, we plot the upper bound for the energy scale at which the transition would most likely happen. Note that for the appropriate mass range the energy scale is much larger than that from the early universe, indicated by the *green* horizontal line, and therefore the transition is very likely to happen in the early universe.

and coordinates $x \to \sqrt{\lambda}^{-1} x$ we find that the Euclidean action takes the form, $S_E = g(b/r_h)/\lambda$, for some function g.

After varying α and β for different values of the black hole to instanton size ratio b/r_h, we have obtained an upper bound on the current probability of decay for the false Higgs vacuum. For instanton scales similar to the size of the black hole horizon, $\mathcal{O}(b/r_h) \sim 1$, we have found a tremendous increase of the decay probability to the false vacuum; in some cases, the decay occurs almost instantly. Using the values of λ estimated by Butazzo *et al.* , we bound the Euclidean action S_E as we vary b/r_h and, by for choosing the instanton size $b \sim 10^{13}$GeV, we bound the decay time for different black hole masses m_{BH} (see Figure 2). While our result depends on the instanton size b, we have chosen $b \sim 10^{13}$GeV in order to obtain field displacements within the range $10^{12}\text{GeV} \ll \phi \ll 10^{19}$GeV and obtain a possible

Our results thus imply that once a single evaporating black hole in our past light-cone probes the mass range presented in Figure 2, the Universe will almost instantly decay to the true vacuum. Note that since the discovered mass range is much smaller than the expected masses of primordial black holes, our result thus places great tension between the standard model and primordial black hole physics.

4 Metric Backreaction and Thermal Effects

Generally, one should also consider the backreaction of the Fubini instanton on the Schwarzschild metric, similar to the way in which Lee and Weinberg *et al.* [7] considered such a backreaction in flat-space. Our variational method once again proves effective, as the backreacted solution is bounded by the solution without backreaction, which in turn is bounded by the action that we computed using different field profiles,

$$S_{E,\text{backreaction}} \leq S_{E,\text{no backreaction}} \leq S_{E,\text{variational}} \tag{11}$$

The inequality on the left is valid as long as the change in the size of the black hole horizon due to backreaction is small [1]. If this is not the case, on the BH horizon, the boundary conditions for our chosen field configurations (10) and those for the actual bounce solution may vary significantly. Consequently, it does not follow by definition that the action corresponding to field configurations that we have chosen is greater than the action of the real bounce solution. On the other hand, for the range of field displacements which we are interested in, the energy stored by the instanton is much smaller than the energy of the black hole. This qualitatively indicates that it is unlikely that the black hole radius will change sufficiently so that the inequalities in Eq. (11) are inapplicable.

Due to the Hawking radiation emitted by the black hole, one also has to consider thermal corrections for the instanton solution. The results of Cheung and Leichenauer [8] indicate that such corrections have the potential to stabilize the Higgs vacuum, therefore making it possible to get an unrealistic result using our variational method. By dimensional analysis, one can see that thermal effects come into the Lagrangian through terms such as $\lambda_T T^2 \phi^2$. While these corrections are important for small field displacements, for large field displacements for which the quartic coupling of the Higgs field goes negative, $\lambda_T T^2 \phi^2 \ll \lambda \phi^4$. As we have seen that the probability of decay transition is maximized when the size of the instanton is of the same order, $O(r_s) \sim O(b)$. In such a case, the temperature of the black hole is given by $T \sim 1/(4\pi b)$. Comparing the thermal term that appears in the Lagrangian to the quartic term we find that,

$$\lambda_T T^2 \phi^2 \ll \lambda \phi^4 \Longleftrightarrow \frac{\lambda_T}{128\pi^2} \ll 1 \tag{12}$$

[1]Smaller than the size of the black hole itself.

Assuming no fine-tunning, the dimensionless thermal coupling constant should be small, $\lambda_T \ll 1$. Thus, the thermal effects due to the black hole are much smaller than the metric distortion effects on the instanton.

5 Discussion

Assuming that the standard model is valid up to energies of 10^{12}GeV, the results we presented in this paper place constraints on primordial black hole physics. We identify three different possibilities:

- Primordial black holes have not formed in the early universe, which poses constraints on some cosmological models.

- All black holes in our past light-cone have yet to evaporate down to the dangerous mass range. Using the estimated age of our universe [9], all primordial black holes which had initial masses $m_{0,BH} < 10^{12}$kg should have already evaporated [10]. This implies that primordial black holes should have been created with masses greater than 10^{12} kg [2]. In such a scenario, the universe might transition to the true vacuum in the near future.

- Black holes do not decay completely, but instead form remnants with masses greater than $m_{BH} > 0.17$ kg. Given our current understanding of black hole physics, it is unlikely that we will find BH remnants with such high masses in our universe.

The initial mass $m_{0,BH}$ of the primordial black hole will be restricted by the size and energy density of the Universe at the time of creation. In particular, the largest black hole that can exist in the Schwarzschild-de Sitter metric in the early universe corresponds to the Nariai black hole, whose event horizon has the same radius as the cosmological horizon. Using the Nariai bound, we find that any black hole created when the universe was at a temperature $T > 10^{10}$GeV should have already evaporated and caused the transition to the true Higgs vacuum. If PBHs are formed due to gravitational perturbations from inflation, most of them had to be created shortly after reheating [11]. This places a bound on the reheating temperature, $T_{\text{rh}} < 10^{10}$GeV. On the other hand, PBHs that are not formed during reheating but rather in the quark-hadron

[2] We worked under the assumption that primordial black holes are sufficiently distant from each other at the time of nucleation such that the instanton forms around the center of a single PBH.

phase [12] or in TeV quantum gravity scenarios [13] have much higher expected initial mass. Since we do not expect these PBHs to have evaporated yet, a transition to the true vacuum would be possible in the near future.

On the other hand, the metastability of the Higgs vacuum assumes that the standard model is valid up to energies smaller than 10^{12} GeV. The discovery of new particles, such as those predicted in SUSY models, have the potential to stabilize the Higgs vacuum. If primordial black holes did exist in our past light-cone, our results indicate that such undiscovered particles have to exist. While our calculations apply to the standard model, the methods we used could also constrain scenarios beyond the SM that still have a metastable vacuum.

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